

Functional linear models for scalar responses

A model for total annual ...

Estimating $\beta(s)$ with a ...

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- With functional responses and multivariate independent variables we could estimate the regression coefficient functions without necessarily needing to use roughness penalties.
- The same with functional responses, functional independent variables and the concurrent model.
- Now we look at a scalar response predicted by a functional independent variable, and discover that a roughness penalty or *regularization* is indispensable.

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1. A model for total annual precipitation

- Let $y_i = \text{LogPrec}_i$ be the logarithm of total annual precipitation at weather station i .
- Here is our model:

$$\text{LogPrec}_i = \alpha + \int_0^T \text{Temp}_i(s) \beta(s) ds + \epsilon_i .$$

- We can think of the function values $\text{Temp}(s)$ associated with each fixed s as a separate scalar independent variable.
- If so, we have enough fitting power at our disposal to fit any number of responses, and certainly only 35 of them.

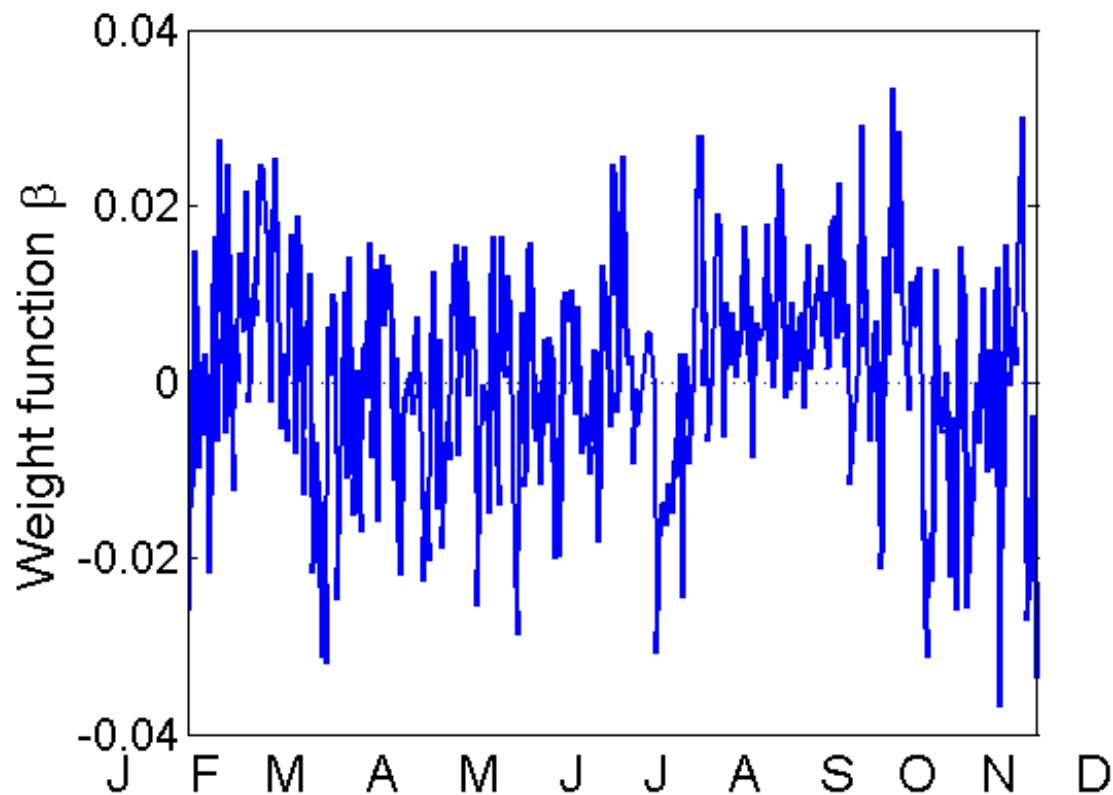
A bad idea

- If we use the discrete daily temperature averages, we have 365 plus 1 for constant α independent variables to fit 35 responses.
- Using the Moore-Penrose generalized inverse to keep us out of trouble, we get the following estimate of β .

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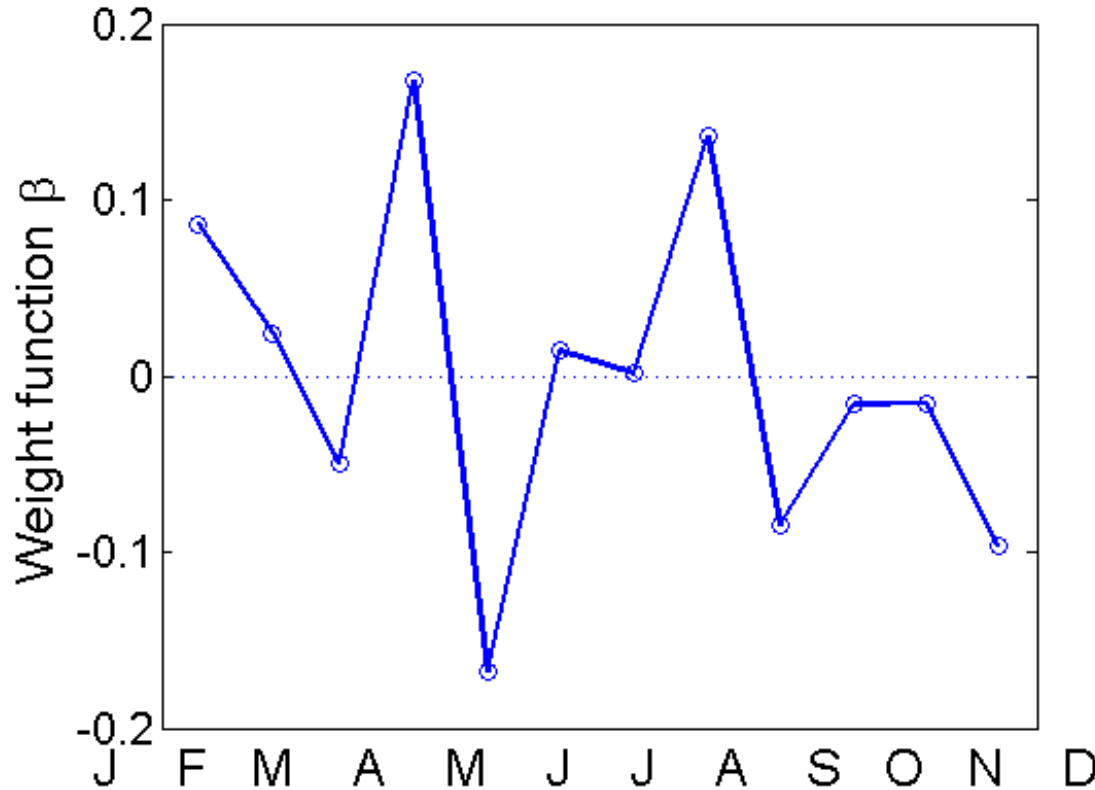
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Using only monthly values doesn't help much.



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2. Estimating $\beta(s)$ with a roughness penalty

- We could impose smoothness on $\beta(s)$ by expanding it in terms of a small number (< 35) of basis functions.
- Using a roughness penalty, however, gives us continuous control over smoothness and other advantages.
- Here is the penalized least squares criterion:

$$\begin{aligned} \text{PENSSE}_{\lambda}(\alpha, \beta) = & \sum_{i=1}^N [y_i - \alpha - \int z_i(s) \beta(s) ds]^2 \\ & + \lambda \int [L\beta(s)]^2 ds, \end{aligned}$$

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Choosing a roughness penalty

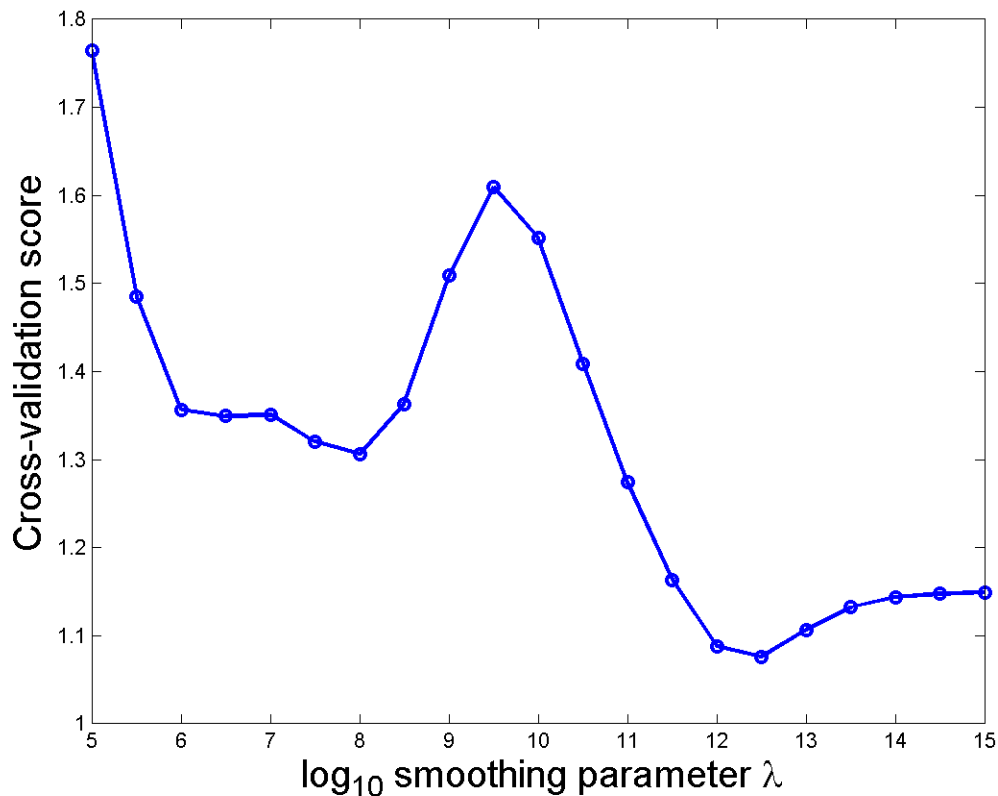
- Let's penalize *harmonic acceleration* because we want $\beta(s)$ to be periodic:

$$L\beta(s) = \left(\frac{2\pi}{365}\right)^2 D\beta(s) + D^3\beta(s)$$

- We choose the smoothing parameter λ by minimizing the cross-validation criterion.
- Let $\alpha_\lambda^{(-i)}$ and $\beta_\lambda^{(-i)}$ be the estimates using all the responses except y_i .
- The criterion to be minimized is

$$\text{cv}(\lambda) = \sum_{i=1}^N [y_i - \alpha_\lambda^{(-i)} - \int z_i(s) \beta_\lambda^{(-i)}(s) ds]^2$$

A plot of $CV(\lambda)$



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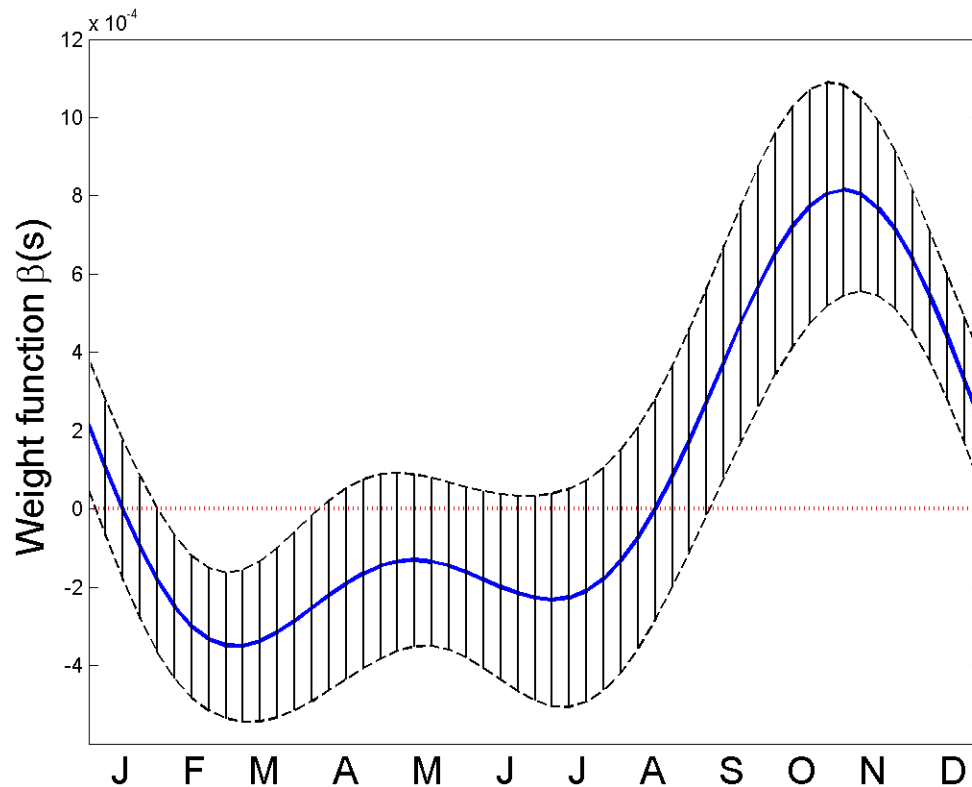
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$\beta(s)$ for $\log_{10} \lambda = 12.5$



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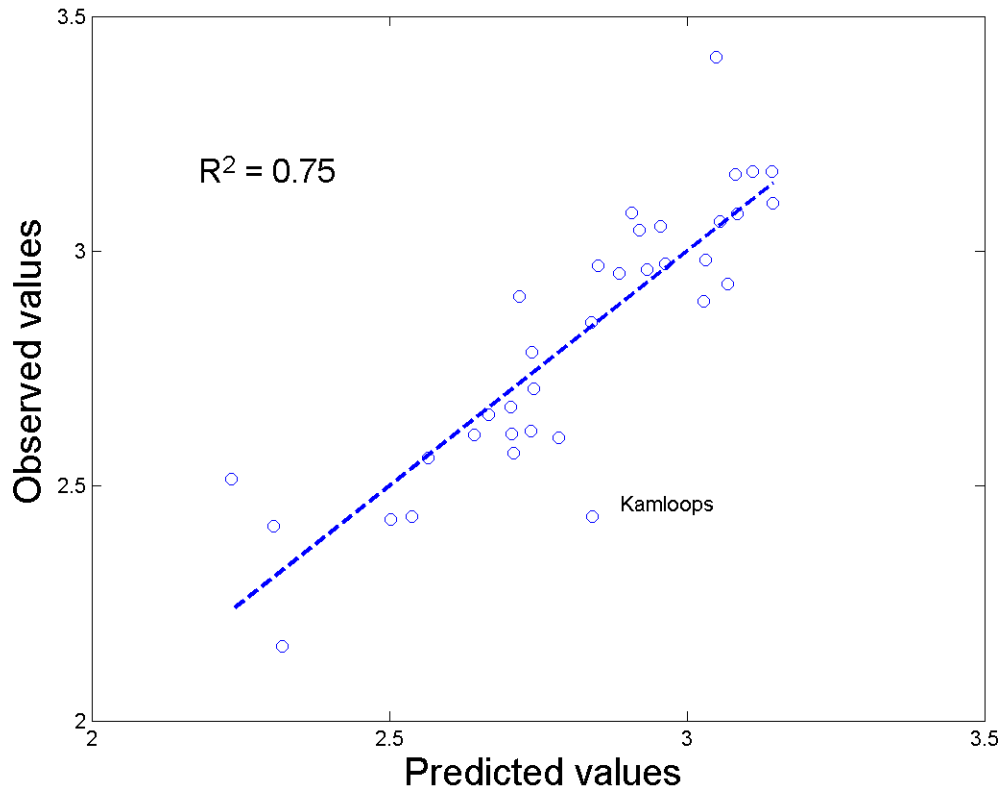
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A plot of the fit



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3. Summary

- Either dimension reduction or regularization is essential when the dimensionality of the covariate exceeds the dimensionality of the response.
- Functional covariates for scalar responses has been heavily researched.
 - The group STAPH at U. Toulouse uses functional PCA to reduce covariate dimensionality. Go to www.lsp.ups-tlse.fr/FP/Ferraty/staph.html to learn more.
 - Gareth James, the group at the U. of Granada, and Doug Clarkson at Insightful Corp. have introduced functional covariates into the generalized linear model.

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