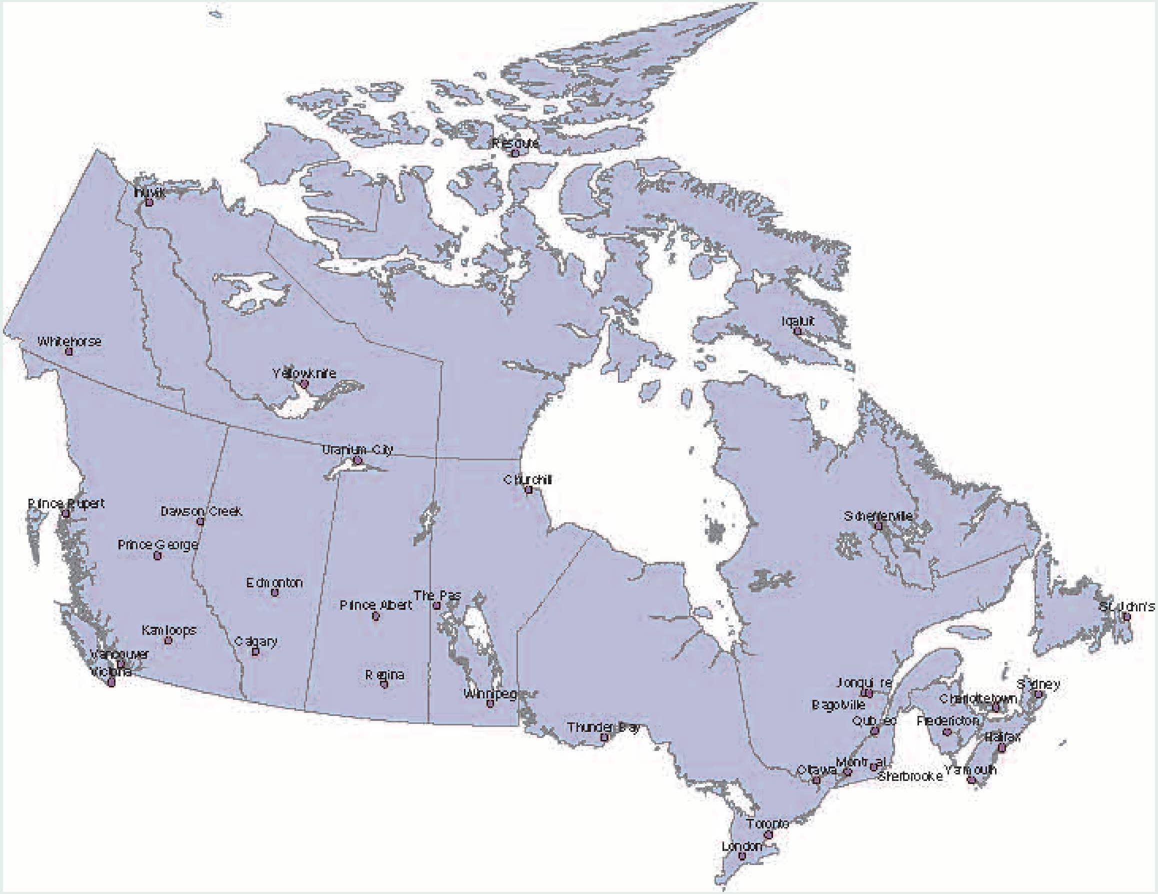
An Overview of the Functional Linear Model

We want to see where these lectures on the functional linear model will go.

- Four functional linear models for the daily weather data.
- A functional ANOVA for precipitation.
- Predicting total annual precipitation from the temperature profile.
- Predicting today's precipitation from today's temperature.
- Predicting the entire year's precipitation from the year's temperature profile.
- A short–term feed–forward model for precipitation.
- A more general perspective.
- Predicting precipitation dynamics: a differential equation
- The idea of a linear model reviewed.

The average Canadian weather data

- 35 Canadian weather stations selected to cover the country.
- Daily temperatures (0.1 degrees Celsius) and precipitations (0.1 mm) averaged over the years 1960 to 1994. (Feb 29th combined with Feb. 28th).
- Canada divided into Atlantic, Continental, Pacific and Arctic weather zones.



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1. A functional analysis of variance

- Does the precipitation profile vary from one weather zone to another?
- We have a number N_g of weather stations in each climate zone $g = 1, \ldots, 4$, and
- the model for the mth temperature function in the gth group, indicated by $\texttt{Prec}_{mg}$, is

$$\texttt{Prec}_{mg}(t) = \mu(t) + \alpha_g(t) + \epsilon_{mg}(t).$$

- $\mu(t)$ is the grand mean function, summarizing precipitation for all of Canada.
- $\alpha_g(t)$ is the functional effect of being in weather zone g
- In order to fix zone effects, we require that

$$\sum_g \alpha_g(t) = 0 \text{ for all } \; g$$

2. A scalar response and a functional independent variable

- The response is the log total annual precipitation

$$\texttt{PrecTot}_i = \int_0^{365} \texttt{Prec}_i(t)\, dt$$

- The model is

$$\log(\texttt{PrecTot}_i) = \alpha + \int_0^{365} \texttt{Temp}_i(s)\beta(s)\, ds + \epsilon_i\,.$$

- But here we have a real problem. How to avoid overfitting the 35 scalar observations?
- We'll use regularization or roughness penalties on the estimated regression functions.

3. A functional response and a functional independent variable

This is a big topic, and breaks down into several useful special versions.

3.1. The concurrent functional model

- We might only use the temperature at the same time $s = t$ because we imagine that precipitation now depends only on the temperature now.
- Our model is

$$\texttt{Prec}_i(t) = \alpha(t) + \texttt{Temp}_i(t)\beta(t) + \epsilon_i(t)$$

- We might call this model *concurrent* or *point-wise*.
- Should we use regularization to force β to be smooth in t?

3.2. The annual or total model

- We may prefer to allow for temperature influence on $\mathtt{Prec}(t)$ to extend over the whole year.
- The model expands to become

$$\mathtt{Prec}_i(t) = \alpha(t) + \int_0^{365} \mathtt{Temp}_i(s)\beta(s,t)\,ds + \epsilon_i(t)$$

- The value $\beta(s,t)$ determines the impact of temperature at time s on precipitation at time t.
- We need roughness penalties for variation in both s and t

3.3. The limited–term feed–forward model

- it may be that what counts is whether the temperature has been falling rapidly up to time t. The model expands to

$$\texttt{Prec}_i(t) = \alpha(t) + \int_{t-\delta}^{t} \texttt{Temp}_i(s)\beta(s,t)\,ds + \epsilon_i(t)$$

- Here δ is the time lag over which we use temperature information.
- Now β is only defined over the somewhat complicated trapezoidal domain: $t \in [0, 365], t - \delta \le s \le t$.

3.4. The local influence model

- Finally, we may open up the model to allow integration over s within a t-dependent set Ω_t.
- The model may therefore be

$$\texttt{Prec}_i(t) = \alpha(t) + \int_{\Omega_t} \texttt{Temp}_i(s)\beta(s,t)\,ds + \epsilon_i(t)$$

4. Predicting derivatives

- When the response is a derivative, then there is the potential for the function itself to be a useful covariate.
- The concurrent linear model

$$D\texttt{Prec}_i(t) = \texttt{Prec}_i(t)\beta(t) + \epsilon_i(t)$$

  is a *homogeneous first order linear differential equation* in precipitation.

- If we also include an influence of temperature,

$$D\texttt{Prec}_i(t) = \texttt{Prec}_i(t)\beta_0(t) + \texttt{Temp}_i(t)\beta_1(t) + \epsilon_i(t),$$

  the equation is said to be *forced* or *nonhomogeneous.*

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5. What exactly makes a model linear?

- We see that the functional linear model has a lot more variants than it's poor multivariate cousin.
- We will need to look at a definition of a linear model that encompasses these models and many others.