Parameter Flows: A Functional Approach to Estimation and Inference

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A parameter flow is an old idea that may have a new future. Choose a fixed point in parameter space, θ_0, and follow the path that improves a fitting criterion $F(\theta|Data)$ as rapidly as possible at each point θ until it reaches the optimizing value $\hat{\theta}$. Techniques for following this *parameter flow line* are among the oldest known optimization algorithms. The flow is defined by a first order differential equation, and its solution by modern numerical methods is now fast and stable. In the case of linear least squares (LLS) problems, the differential equation is linear, and the solution $\theta(t)$ is available analytically.

The idea is to search along the flow line for parameter estimates that have better mean square error (MSE) properties than the optimal estimate. In the LLS case, $MSE(t), Var(t)$ and $Bias(t)$ anywhere on the flow are computable analytically. It is shown that we can often find estimates $\theta(t)$ with impressively improved $MSE(t)$, and especially when working with small sample sizes, noisy data, and/or an intelligent initial value θ_0.

A number of real data examples and some systematic results are offered to show that the MSE of the so-called optimal estimate can often be halved by finding a value along the flow line short of the optimum. Some easily implementable methods for identifying these values are discussed. Implications for hypothesis testing and other inferential problems are also considered.